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Application of the information theory to the description of the phosphorus compounds reduction at a sewage treatment plant

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Abstract

The objective of the study is to show that the correlation and spectral analysis of the stochastic input and output signals obtained from technological systems can be used for mathematical description of the process on the basis of the information theory. The statistical analysis and the new quantum informational criterion is proposed. These analyses may be successfully applied to the description of the chemical engineering systems [V. Zhukov, V. Mizonov, P. Filitchev, S. Bernotat, The modeling of grinding processes by means of the principle of maximum entropy, Powder Technol. 95 (1998) 248; S. Masiuk, R. Rakoczy, The entropy criterion for the homogenisation process in a multi-ribbon blender, Chem. Eng. Process., 45 (6) (2006) 500]. In this paper, the informational approach is used for evaluation of the informational power characteristic of the phosphorus reduction in the raw sewage in the functioning sewage treatment plant. These considerations permit to formulate the quantum information criterion for the new predicting system (NS) and the subsystems (functioning sewage treatment system, OS) and (additional system, AS).

Keywords: Correlation and spectrum analysis; Informational entropy; Quantum information

1. Introduction

One of the traditional approaches to mathematical modelling of the chemical processes is based on the correlation and spectrum analysis of the input and output stochastic signals. These signals may be found from measurements in real functioning systems as the time cross-section of the discrete stochastic processes. In the case of the industrial database, the discrete stochastic function describing the stochastic nature of a process may be defined as the finite collection of the experimental data obtained for the different sampling moments. Another way to avoid the difficulties connected with the correlation analysis is reducing the discrete stochastic process to the representative discrete stochastic function. In this case it must be assumed that the input and output databases are ergodic. In many cases this assumption is a priori accepted. This statement permits to simplify the analyses of the experimental database.

The interesting alternative to the classical methods is the statistical description of the chemical engineering processes by means of the information theory. Unfortunately, in this case the knowledge of the probability density distribution for the each time cross-section of the stochastic process is required. It is important problem in the mathematical analysis of the chemical engineering dynamic systems. Moreover, the connected informational theory and the statistical analysis may be useful for the optimisation of chemical engineering processes. This theory may be perfectly adapted to the dynamical description of the process of reduction of phosphorus in the functioning sewage treatment plant.

The number of references to the works, which illustrates how the information theory and the informational entropy may be applied to the description of real chemical engineering systems, is not very extensive. In the recent papers the Shannon's entropy [1] has been applied to the description of some of practical applications of the concept: for quantum systems [2]; for dynamical systems [3]; for fuzzy sets [4]; for the process of self-organisation in an open unsteady-state non-equilibrium system [5].

The quantitative measurement of the information came from the study of communication; then this fundamental concept and method has been widely applied in many fields such as statistics, physics, chemistry, biology, lifescience, psychology,

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Nomenclature

H	entropy of information (Eq. (1)), bit
I	quantum information (Eq. (2)), bit
$I^{\text{superscript}}$	informational criterion, bit
$R_{H^{\text{supscript}}H^{\text{supscript}}}$	informational correlation function, bit^2
$S_{H^{\text{supscript}}H^{\text{supscript}}}$	informational spectral density function
$\lvert W^{\text{superscript}}(j\omega)\rvert^2$	informational power characteristic

Greek letters

τ	correlation domain
ω	frequency time domain

Superscripts

AS	additional system of the sewage purification
NS	new system of the sewage treatment with the additional purification process
OS	functioning sewage treatment system
‘	raw sewage
“	treated sewage
*	required treated sewage
•	estimator

cybernetics, computer science, economics, operation research, linguistics, philosophy [6–9]. Also, the concept of the informational entropy has been effectively applied in some fields of chemical engineering. The mathematical model of mixing of liquids based on the entropy approach was proposed by Ogawa and Ito [10]. Further, Ogawa et al. [11,12] developed the concept of informational entropy and its application to description of other mixing processes. The new original informational description of the process of segregation and homogenisation during the blending process based on the concept of informational entropy was proposed in Refs. [13,14]. The information theory was successfully used to describe the porous media structure [15] and to identify the clusters in magnetic dispersion [16].

Increasing demand of water supply and difficulties in the conventional water resources development create several technological problems for industries and domestic water supplies. Therefore, new technologies of water treatment could contribute to the solution of the most of existing problems. An important problem in designing of a sewage treatment plants is taking into consideration all actual features of experimental database for a product. The dynamical behaviour of the plant may be described by using the informational approach, in which the informational entropy of the input and output concentration distributions is used as the informational measure of the phosphorus concentrations in the raw and sewage treatment, respectively. Then, the entropy play the role of the informational characteristic of the stochastic process state, and it is adopted for the description of the random nature of the process of reduction of suspension in the raw sewage for a functioning treatment plant [17]. This approach may be successfully used to the description of the stationary linear dynamics of the sewage treatment plant using a simple model based on the information theory. This paper presents the advantage of the informational entropy for the purpose of mathematical description of environment protection processes with a reasonable level of the model complexity.

2. Experimental

The experimental industrial data on the phosphorus concentration in the raw and the treated sewage were obtained at a typical mechanical–biological sewage treatment plant of the municipal sewage in the West-Pomeranian region of Poland. The schematic block diagram of the sewage treatment plant is graphically presented in Fig. 1, where are given the sampling points of the phosphorus concentrations in the raw and treated sewages (inlet and outlet signals).

These databases are realistic for the actual situation of the process of reduction of the phosphorus in the sewage. The measurements of phosphorus concentration were performed according to the common procedure required by the Polish Standard-C-04537-14. The data are presented in form of the discrete stochastic functions of the input signals (see Fig. 2) as well as for the output signals (see Fig. 3) obtained simultaneously. The data points of the phosphorus concentration in the raw and treated sewages, that are presented in these figures, were measured synchronously at the specific time interval (day) at the normally functioning treatment sewage plant. The collections of the data are arranged in the way that allows simply calculating the grand totals, averages, ranges and distributions. Unfortunately, the accuracy and precision of the industrial measurements at the plant are unknown.

3. Experimental data treatment

3.1. The informational entropy

An important method of mathematical modelling of the dynamic systems is the statistical correlation technique based on the dependence between the stochastic input and output signals. This technique does not require any particular form for the test signal but is very sensitive to the noise of them. Even in the case, when the structure of a system is not very complex, and the system can be subdivided into the subsystems, that may be treated as separate units, the direct sinusoidal-testing, step-testing and pulse-testing signals may be used to analyse these subsystems without destructive influence on the efficiency of the whole system. Although the statistical approach to evaluation of mathematical models for dynamic systems is considered to be cumbersome, it allows the adequate analysis of the random signals and gives the satisfactory description.

In the case of the sewage treatment plant, the structure of the system is complex because the mechanical and the biological purification processes are interconnected. Therefore, the individual subsystems with the elementary processes cannot work separately, and it means that the processes of reduction of the pollutions in the sewage should be treated as the united chemical engineering process. In this case, the actual dynamic behaviour

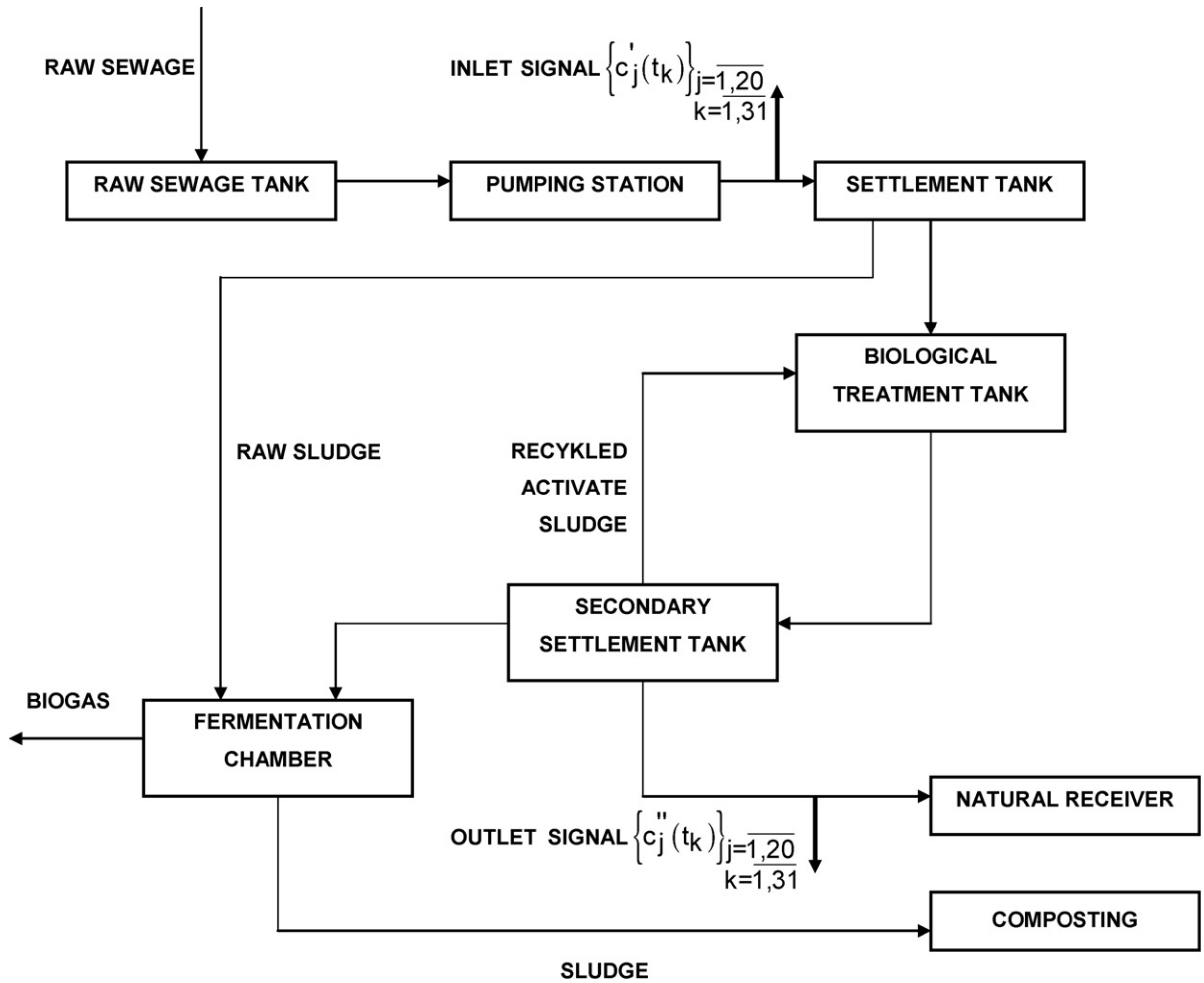

Fig. 1. Schematic block diagram of the typical mechanical–biological sewage treatment plant (where "*j*" is number probes in the sampling process; "*k*" is sampling moment).

of this system may be detected only by using the stochastic database of the input and output signals obtained simultaneously from the functioning sewage treatment plant. However, for the very complex system, that is disturbed by uncontrolled random input, it is difficult to propose the structure of the block diagram and evaluate model parameters of the system in the time and frequency domains.

Traditionally, some state variables for a random signal are used in the statistical analysis. The new alternative approach to the description of the system is based on the information

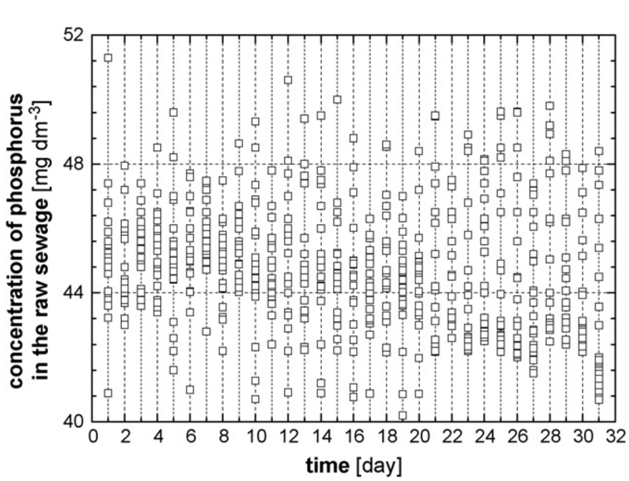

Fig. 2. Variation with time of phosphorus concentration in the raw sewage.

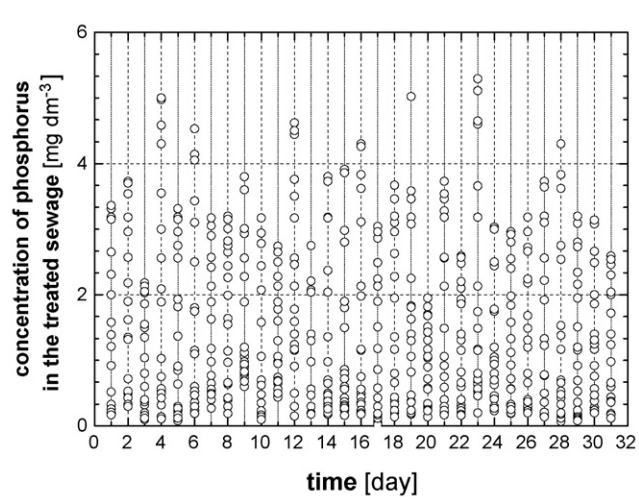

Fig. 3. Variation with time of phosphorus concentration in the treated sewage.

theory and the Shannon's entropy. Application of this theory to the statistical technique gives the full pattern of the process behaviour in the area of the classical and informational strategy for the system identification and the automatic control.

On the basis of the probability analysis at the each time cross-section of the stochastic processes (Figs. 2 and 3), the informational entropy is defined as the characteristic of the random values collection. The input and output informational entropy can be characterised by the following equation:

$$H(\{\Delta c^{\rm sign}\};k) = -\sum_{m=1}^{M} w(\Delta c_m^{\rm sign};k)\log_2 w(\Delta c_m^{\rm sign};k),\ [\text{bit}] \qquad (1)$$

where $w(\Delta c_m^{\rm sign};k)$ is the probability function for the phosphorus concentration interval, $\Delta c_j^{\rm sign}$, at the k-moment, $\Delta c_m^{\rm sign} = c_{m\,\max}^{\rm sign} - c_{m\,\min}^{\rm sign}/M$ the interval of the phosphorus concentration, $\Delta c_m^{\rm sign} \in \{\Delta c_m^{\rm sign}\}_{m=\overline{1,M}}$ the phosphorus concentration interval for the raw (′), or the treated (″) sewage, m the number of phosphorus; concentration interval in the collection of the sampling data for the each time cross-section of the stochastic process, k the number of the discrete time cross-section of the stochastic process, and M is the number of phosphorus concentration intervals (in this work $M = 10$).

It is well known that the significant simplification in calculation of the informational entropy can be reached by assuming that the random input and output industrial signals are described by a typical distribution. Then, it is possible to easily execute four simple and relatively different types of statistical characteristics describing the industrial database of the phosphorus concentration in the raw and treated sewage: the mean values, the standard deviation, the asymmetry and the kurtosis. These specified statistical characteristics in the form of the scatter diagrams were used to estimate the correlation between the actual distributions of the experimental data and the chosen typical distribution. These characteristics are not included in this paper. It was found that the mean values and standard deviations for the raw and treated sewage are practically independent of time. Hence, the collections of the data on phosphorus concentration obtained from the sections of the database at sampling times represent the discrete stationary stochastic processes without trend with respect to the mean values. The analysis of the asymmetry and the kurtosis showed that the values of the asymmetry and the kurtosis were mainly located in the intervals corresponding to the normal distribution. As follows from these remarks, the practical distribution for the input and output of the industrial database at each time section can be treated as the stochastic process with the normal distribution.

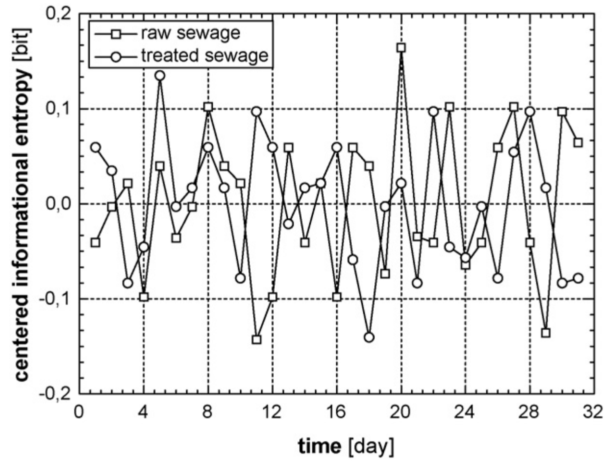

Fig. 5. The centered values of the informational entropy for the phosphorus concentration at different time intervals.

Fig. 4 shows the example of the typical comparison of the experimental cumulative distributions (points with thin line) for the raw and treated sewage and the calculated cumulative probability for the normal distribution (solid lines) as the confirmation of the above statement.

Application of Eq. (1) to the industrial databases for the input and the output phosphorus concentration allows to calculate the values of the informational entropy at each time section of the stochastic process. The variation of the centred values of the informational entropy for the raw and treated sewage is shown in Fig. 5.

3.2. The quantum information

When formulating and solving the identification problem as the optimisation problem, it is important to formulate an appropriate criterion. There exist different criteria to be optimised in

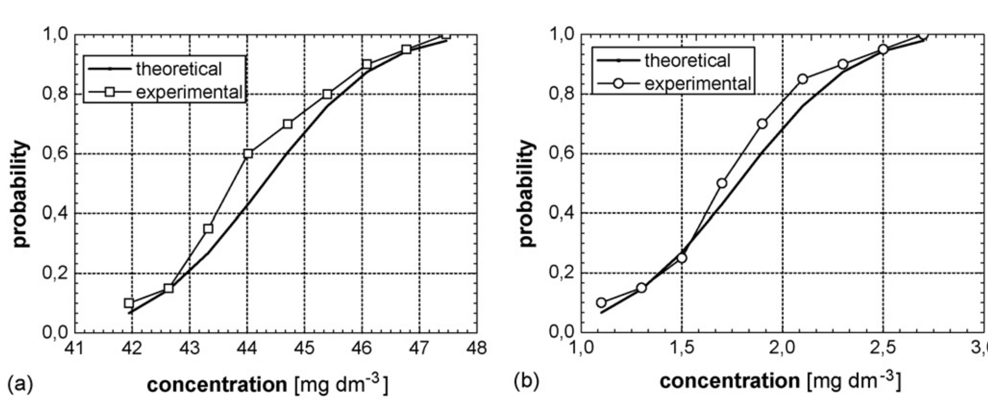

Fig. 4. Comparison of the cumulative normal probability distributions (solid line) and the experimental cumulative distributions (points with thin line) for the concentration interval in the raw sewage (a) and treated sewage (b).

mathematical calculation. One of them is the quantum information, which is widely used in the study of signals transmission. This criterion is formulated at the time and frequency domains, and it may be applied in many probabilistic situations of the industrial chemical engineering system dynamics. In particular, the information criterion makes it possible to define the relation between the identification and the control system. This approach is very natural, particularly if the industrial database contains the stochastic input and the output signals. Treating the informational entropy as the state variable, it is possible to define the correlation function and the spectral density. It is clear that the dynamic analysis is more precise if the process behaviour is formulated as the information problem. The quantum information may be adequate to the identification problem for the system if the probabilistic characteristics of the input and output signal are known, or may be easily calculated. The quantum information at the kth moment of the time cross-section of the discrete stochastic process, $I^k_{\{c'_j(t_k)\}}(\{c''_j(t_k)\})$, may be characterised by the equality:

$$I^k_{\{c'_j(t_k)\}}(\{c''_j(t_k)\}, k) = H^k(\{c'_j(t_k)\}, k) - H^k_{\{c'_j(t_k)\}}(\{c''_j(t_k)\}, k) \tag{2}$$

As follows from Fig. 5, the centred informational entropies of the phosphorus concentration are the discrete stochastic functions at the time. From the practical point of view, the application of the informational criterion looks highly desirable in order to use the correlation and the spectral analysis. Then, it is possible to obtain general analytical forms of the quantum information in the time domain Eq. (3) and in the frequency domain Eq. (4), respectively:

$$I(\tau) = R_{\overset{o}{H'}\overset{o}{H'}}(\tau) - \frac{R^2_{\overset{o}{H'}\overset{o}{H''}}(\tau)}{R_{\overset{o}{H'}\overset{o}{H'}}(\tau)} \tag{3}$$

$$I(\omega) = S_{\overset{o}{H'}\overset{o}{H'}}(\omega) - \frac{S^2_{\overset{o}{H'}\overset{o}{H''}}(\omega)}{S_{\overset{o}{H'}\overset{o}{H'}}(\omega)} \tag{4}$$

3.3. The correlation and spectral analysis

It is well known that the entropy Eq. (1) is commonly used as the measure of uncertainty in sending information between the source and the receiver. Thus, the entropy is one of the informational state variables. In the chemical engineering processes the calculated entropies for the input and output signals may be used to find the correlation functions on the basis of the statistical correlation technique.

As it was mentioned above, the centred informational entropies for the raw and treated sewage are used as the discrete stochastic functions. From the analytical point of view, these functions can be described in the time domain by means of the standard correlation analysis using the typical estimators. When the stochastic functions $\overset{o}{H'}(k)$ and $\overset{o}{H''}(k)$ are stationary at the time and ergodic, the estimators of the informational auto-correlation function for the raw sewage, $R^{\bullet}_{\overset{o}{H'}\overset{o}{H'}}(k)$, the informational auto-correlation function for the treated sewage, $R^{\bullet}_{\overset{o}{H''}\overset{o}{H''}}(k)$, and the informational cross-correlation function between the raw and the treated sewages, $R^{\bullet}_{\overset{o}{H'}\overset{o}{H''}}(k)$ have the following forms:

$$R^{\bullet}_{\overset{o}{H'}\overset{o}{H'}}(k) = \frac{1}{N-k}\cdot\sum_{i=1}^{N-k}\overset{o}{H'}(i)\cdot\overset{o}{H'}(i+k) \tag{5}$$

$$R^{\bullet}_{\overset{o}{H''}\overset{o}{H''}}(k) = \frac{1}{N-k}\cdot\sum_{i=1}^{N-k}\overset{o}{H''}(i)\cdot\overset{o}{H''}(i+k) \tag{6}$$

$$R^{\bullet}_{\overset{o}{H'}\overset{o}{H''}}(k) = \frac{1}{N-k}\cdot\sum_{i=1}^{N-k}\overset{o}{H'}(i)\cdot\overset{o}{H''}(i+k) \tag{7}$$

where k is the number of the correlation time interval, i the range variability of the correlation time interval, and N is the total number of the correlation time intervals in the time duration of the informational entropy cantered for the raw and the treated sewage (see Fig. 5).

Using the calculated data shown in Fig. 5 and applying Eqs. (5)–(7) the values of the estimators of the auto-correlation and the cross-correlation functions of the informational entropy for the raw and treated sewages may be obtained. An important problem in mathematical modelling of the concentration reduction in a sewage treatment plant based on the industrial database is the analytical description of the results in the time domain by a family of deterministic functions having the Fourier transform. The approximating function may give a qualitative explanation of the behaviour of the sewage treatment process, and it should be chosen as a simple function describing the wide range of the estimator values with the minimum error. Using the computational software Matlab® the auto-correlation functions and the cross-correlation function of the informational entropy for the raw and treated sewages were approximated. The Fourier transform of the approximated correlation function gives the so-called spectral density representing the link between the time and frequency domains. The analytical description of the correlation functions and the adequate spectral density functions for the informational entropy are given in the following equations for:

- the auto-correlation function and the spectral density for the raw sewage, respectively:

$$R_{\overset{o}{H'}\overset{o}{H'}}(\tau) = 0.0059\cdot e^{-0.055\cdot|\tau|}\cdot\cos(0.33\cdot\tau) \tag{8}$$

$$S_{\overset{o}{H'}\overset{o}{H'}}(\omega) = 0.0059\cdot\left[\frac{0.055}{0.055^2+(\omega+0.33)^2}+\frac{0.055}{0.055^2-(\omega+0.33)^2}\right] \tag{9}$$

- the auto-correlation function and the spectral density for the treated sewage, respectively:

$$R_{\overset{\circ}{H}''\overset{\circ}{H}''}(\tau) = 0.0044 \cdot e^{-0.12\cdot|\tau|} \cdot \cos(0.25\cdot\tau) \tag{10}$$

$$S_{\overset{\circ}{H}''\overset{\circ}{H}''}(\omega) = 0.0044 \cdot \left[\frac{0.12}{0.12^2 + (\omega+0.25)^2} + \frac{0.12}{0.12^2 - (\omega+0.25)^2}\right] \tag{11}$$

- the cross-correlation function and the spectral density between the raw and the treated sewages, respectively:

$$R_{\overset{\circ}{H}'\overset{\circ}{H}''}(\tau) = -0.0014 \cdot e^{-0.015\cdot|\tau|} \cdot \cos(0.37\cdot\tau) \tag{12}$$

$$S_{\overset{\circ}{H}''\overset{\circ}{H}''}(\omega) = -0.0014 \cdot \left[\frac{0.015}{0.015^2 + (\omega+0.37)^2} + \frac{0.015}{0.015^2 - (\omega+0.37)^2}\right] \tag{13}$$

These expressions are quite flexible, and the mathematical description of the process of the reduction of phosphorus concentration in the sewage may be applied to individual cases with the required level of confidence. The graphical interpretation of Eqs. (8)–(13) are shown in Figs. 6–8.

From the calculation of the correction functions was established, that these function are the symmetric functions. In all figures (Figs. 6 and 7) on the position above the correlation function graph for the real time are given the correlation functions in the whole time variation intervals.

3.4. The quantum information criterion

As it was mentioned in Section 3.2, the quantum information may be used for the analytical evaluation of the efficiency of the process phosphorus reduction in the municipal sewage. The actual nowadays degree of the phosphorus reduction in the sewage is not sufficient to meet the rigorous increase of the requirements in the environment protection. The technological conditions at the existing sewage treatment plants are undoubtedly important, but the new regulations require the application of additional sewage purification processes. According to the regulations that are given in the Direction 91/271/EWG [18,19] the output concentration of the phosphorus in the treated sewage must be additionally decreased to the level equal to 75% (for the equivalent inhabitants number in the range from 10,000 to 14,999). The new time cross-sections of the generated discrete stochastic process from the experimental database of the output phosphorus concentration in the treated sewage is presented in Fig. 9.

The informational entropy of the new required output phosphorus concentration (Fig. 9) can be calculated using the basic Eq. (1). This entropy is essential for to describe of the industrial reduction process with the additional sewage purification.

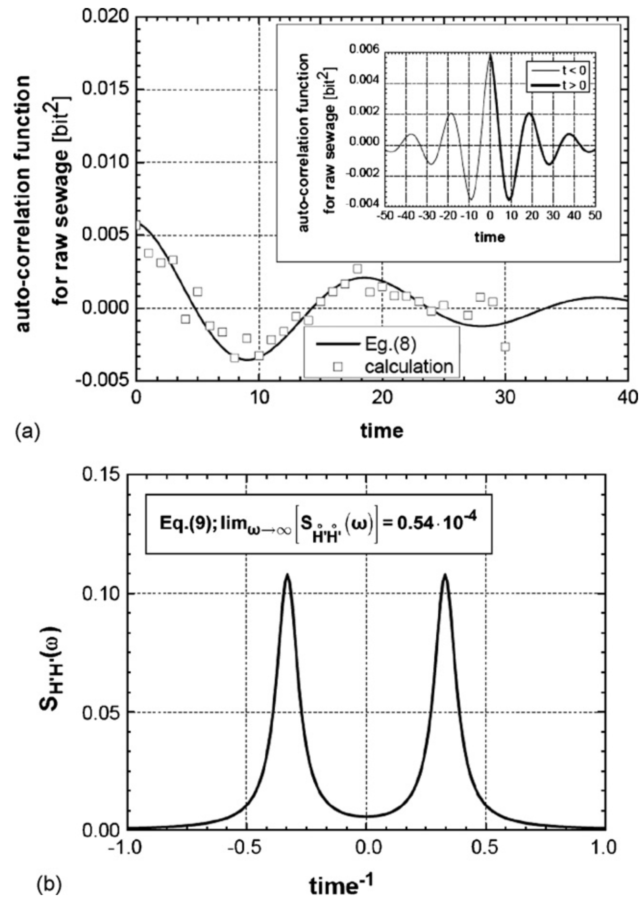

Fig. 6. The auto-correlation (a) and the adequate spectral density (b) functions for the informational entropy of the raw sewage (points are values of estimator; solid line is the analytical approximation of the calculated data).

The centered values of new calculated informational entropy are graphically presented in Fig. 10.

The analytical description of the obtained results may be performed by use of the procedure similar to given in Section 3.3. The estimated values of the required auto-correlation function, $R^{\bullet}_{\overset{\circ}{H}^{*}{}''\overset{\circ}{H}^{*}{}''}(k)$, and the cross-correlation function, $R^{\bullet}_{\overset{\circ}{H}^{*}{}'\overset{\circ}{H}^{*}{}''}(k)$, are computed using the adequate expressions similar to Eqs. (6) and (7). The satisfactory deterministic descriptions for the correlation function and the spectral density of the informational entropy are obtained on the basis of the scatter data of the relevant estimators:

- the new auto-correlation and the spectral density functions for the treated sewage with the additional reduced phosphorus concentration, respectively:

$$R_{\overset{\circ}{H}^{*}{}''\overset{\circ}{H}^{*}{}''}(\tau) = 2.2838 \cdot e^{-0.14\cdot|\tau|} \cdot \cos(0.22\cdot\tau) \tag{14}$$

$$S_{\overset{\circ}{H}^{*}{}''\overset{\circ}{H}^{*}{}''}(\omega) = 2.2838 \cdot \left[\frac{0.14}{0.14^2 + (\omega+0.22)^2} + \frac{0.14}{0.14^2 - (\omega+0.22)^2}\right] \tag{15}$$

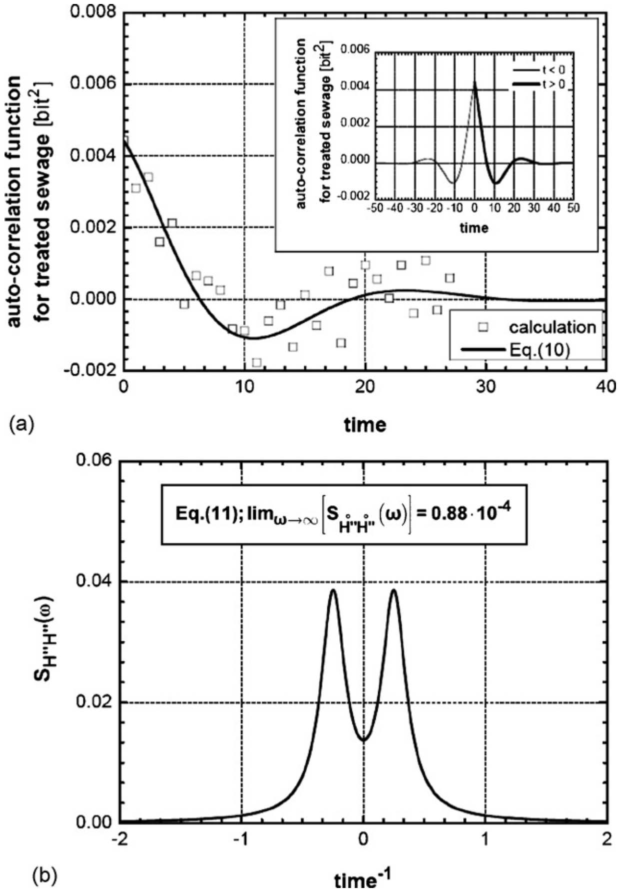

Fig. 7. The auto-correlation (a) and the adequate spectral density (b) functions for the informational entropy of the treated sewage (points are values of estimator; solid line is the analytical approximation of the calculated data).

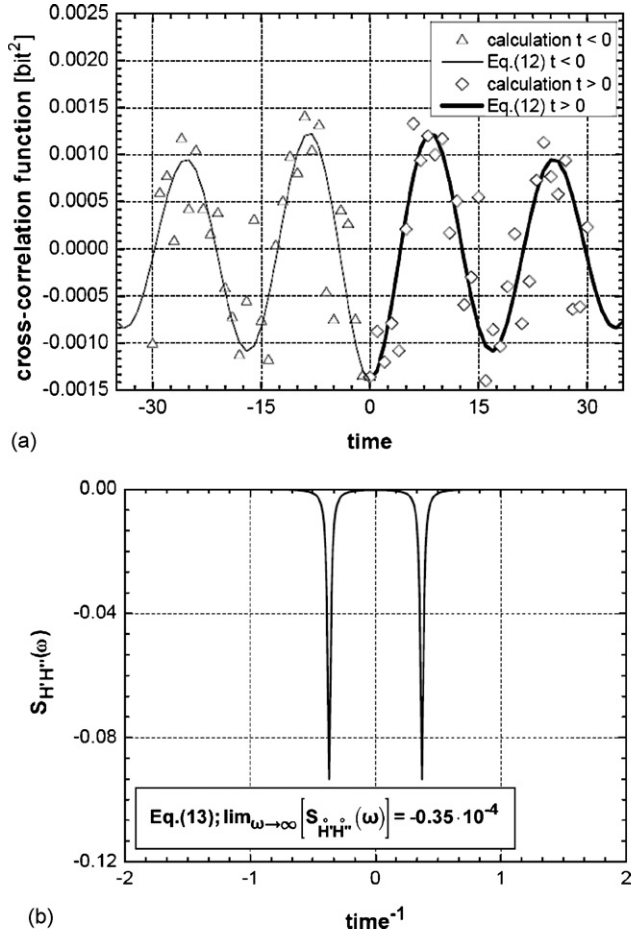

Fig. 8. The cross-correlation (a) and the adequate spectral density (b) functions between the informational entropy of the raw and treatment sewage (points are values of estimator; solid line is the analytical approximation of the calculated data).

- the new cross-correlation and the spectral density functions between the raw sewage and treated sewage with the additional reduced phosphorus concentration, respectively:

$$R_{\overset{\circ}{H'}\overset{\circ}{H^{*''}}}(\tau) = -0.13357\cdot e^{-0.1\cdot|\tau|}\cdot\cos(0.3\cdot\tau) \tag{16}$$

$$S_{\overset{\circ}{H'}\overset{\circ}{H^{*''}}}(\omega) = -0.013357\cdot\left[\frac{0.1}{0.1^2+(\omega+0.3)^2}+\frac{0.1}{0.1^2-(\omega+0.3)^2}\right] \tag{17}$$

The values of the estimators and the approximation functions Eqs. (14)–(17) are shown in Figs. 11 and 12.

The growth of the requirements in the environment protection obliges the functioning sewage treatment plants to improve the efficiency of phosphorus concentration reduction in the municipal sewage. In this case, the preliminary reduction of phosphorus concentration can be reached by using the chemical precipitation process. The most satisfactory result may be reached when the sedimentation and precipitation processes are interconnected. By using the solution of calcium dioxide and addition of carbon dioxide, the phosphorus must be transformed into the indissoluble hydroxyphosphorite.

The mathematical description of the process in the real functioning sewage treatment plant may be done by means of the analysis of the informational power process characteristics. The process reduction of the phosphorus concentration form the raw

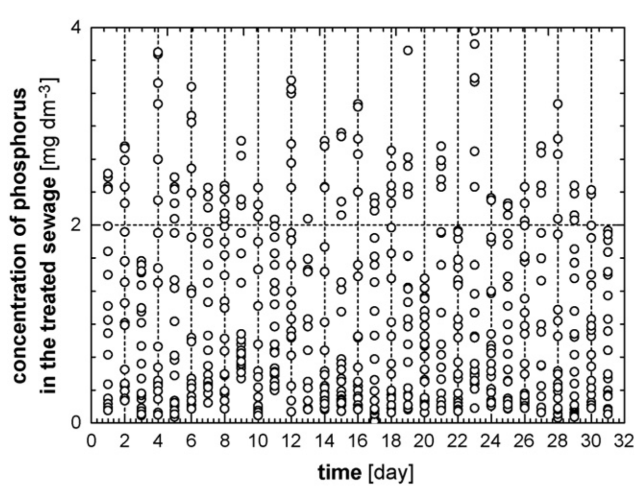

Fig. 9. Temporal variation of the new time cross-sections of the generated discrete stochastic process.

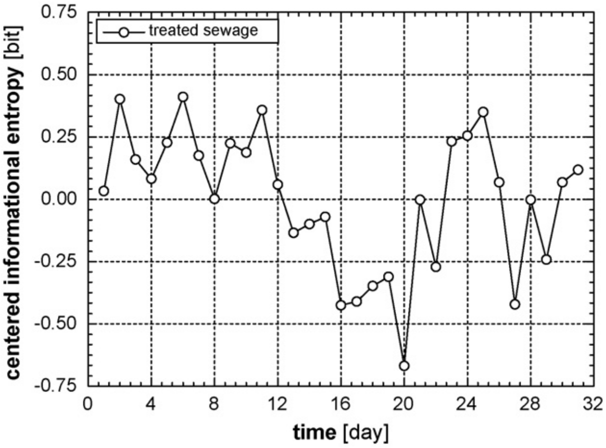

Fig. 10. The centered values of the informational entropy for the industrial reduction process with the additional sewage purification.

sewage may be simulated in the frequency domain using the informational power characteristics, as follows:

- by the use of the additional system (AS) based on the chemical precipitation processes connected with the functioning sewage treatment system (OS),

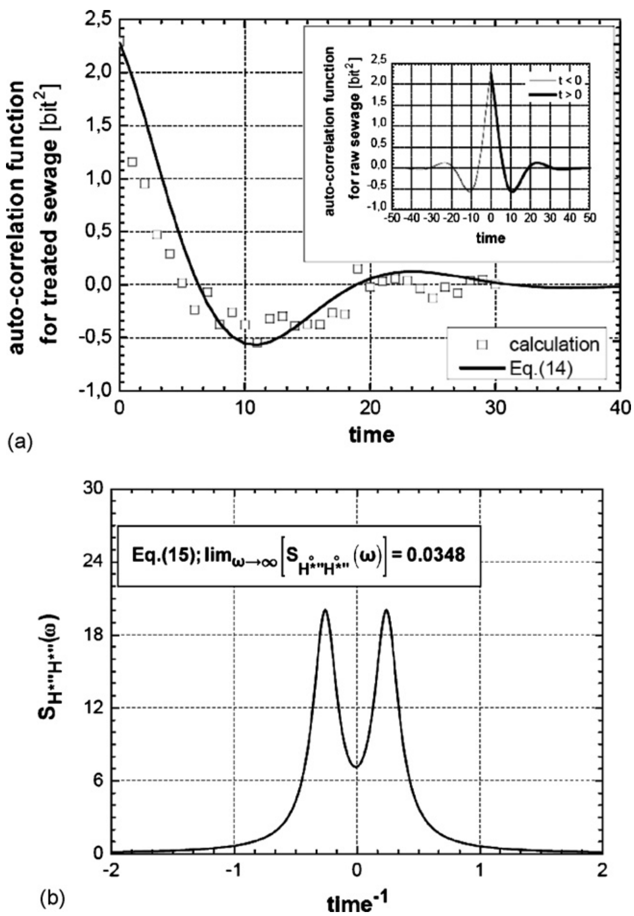

Fig. 11. The auto-correlation (a) and the adequate spectral density (b) functions of the informational entropy for the additional reduced phosphorus concentration in the treated sewage (points are values of estimator; solid line is the analytical approximation of the calculated data).

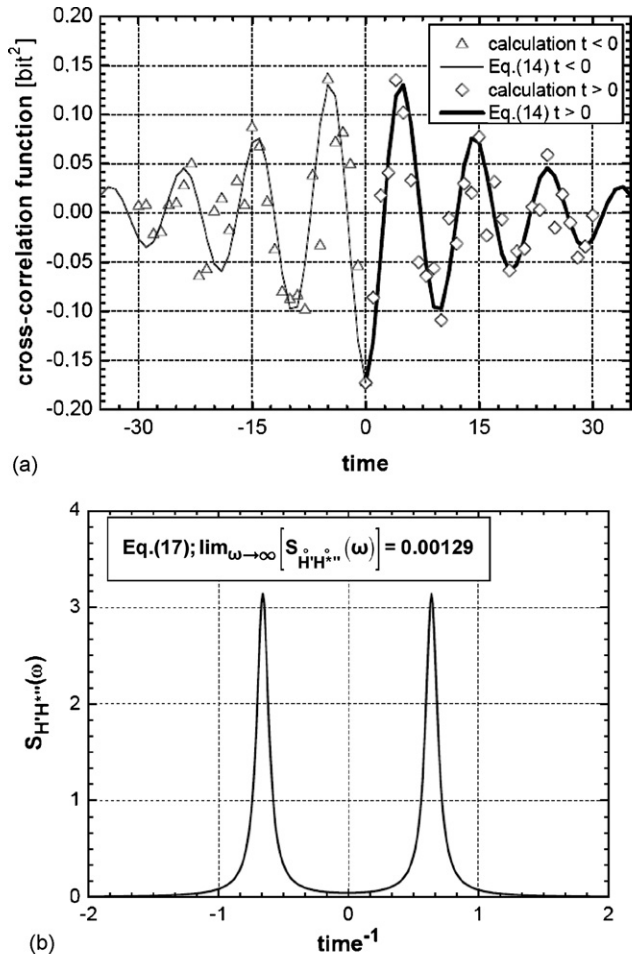

Fig. 12. The cross-correlation (a) and the adequate spectral density (b) functions between the informational entropy of the raw sewage and the treated sewage with the additional sewage purification process (points are values of estimator; solid line is the analytical approximation of the calculated data).

- by the use of the new predicting system (NS) of the sewage purification.

The graphical presentation of the simulation of the processes purification of the raw sewage is illustrated in Fig. 13.

The informational power characteristics in the frequency domain of the functioning sewage treatment, of the additional (AS) and of the new (NS) system shown in Fig. 13 are defined

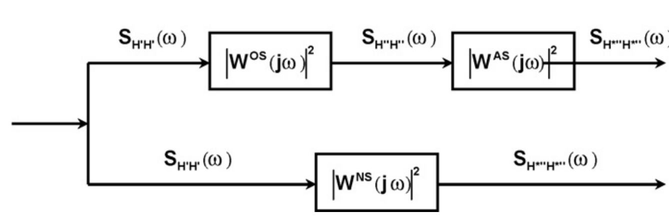

Fig. 13. The scheme of two ways improvement of the phosphorus reduction processes.

as follows:

$$\left|W^{OS}(j\omega)\right|^2=\frac{S_{\overset{o}{H''}\overset{o}{H''}}(\omega)}{S_{\overset{o}{H'}\overset{o}{H'}}(\omega)};\quad \left|W^{AS}(j\omega)\right|^2=\frac{S_{\overset{o}{H^{*''}}\overset{o}{H^{*''}}}(\omega)}{S_{\overset{o}{H''}\overset{o}{H''}}(\omega)};\quad \left|W^{NS}(j\omega)\right|^2=\frac{S_{\overset{o}{H^{*''}}\overset{o}{H^{*''}}}(\omega)}{S_{\overset{o}{H'}\overset{o}{H'}}(\omega)} \tag{18}$$

The informational criterion of the process quality should be based on the quantitative information flow for the system containing the subsystem (OS) and (AS), or for the new system. Employing the information theory, the quantum information in the frequency domain for the functioning sewage treatment subsystem (OS) can be expressed in the form:

$$I^{OS}(\omega)=S_{\overset{o}{H'}\overset{o}{H'}}(\omega)-\frac{S^2_{\overset{o}{H'}\overset{o}{H''}}(\omega)}{S_{\overset{o}{H'}\overset{o}{H'}}(\omega)} \tag{19}$$

The quantum information in the frequency domain after improvement of the process is defined using the similar expression to Eq. (19) and the informational spectral densities (Eqs. (11) and (17)). For the additional subsystem the quantum information is established as:

$$I^{AS}(\omega)=S_{\overset{o}{H''}\overset{o}{H''}}(\omega)-\frac{S^2_{\overset{o}{H'}\overset{o}{H^{*''}}}(\omega)}{S_{\overset{o}{H''}\overset{o}{H''}}(\omega)} \tag{20}$$

Introducing the definition $|W^{AS}(j\omega)|^2$ into Eq. (20) yield:

$$I^{AS}(\omega)=S_{\overset{o}{H''}\overset{o}{H''}}(\omega)-\frac{S^2_{\overset{o}{H'}\overset{o}{H^{*''}}}(\omega)}{S_{\overset{o}{H^{*''}}\overset{o}{H^{*''}}}(\omega)}\cdot\left|W^{AS}(j\omega)\right|^2 \tag{21}$$

On the basis of the predicted system (lower technological line of the block scheme Fig. 13) and the definition of the quantum information given by Eq. (2) the informational criterion in the frequency domain for the new system may be described by the following equation:

$$I^{NS}(\omega)=S_{\overset{o}{H'}\overset{o}{H'}}(\omega)-\frac{S^2_{\overset{o}{H'}\overset{o}{H^{*''}}}(\omega)}{S_{\overset{o}{H'}\overset{o}{H'}}(\omega)} \tag{22}$$

Assumption that the phosphorus concentration in the treated sewages for the new system and for the system containing the subsystems (OS) and (AS) are equal, then the informational quantum criterion has the following relationship:

$$I^{NS}(\omega)=I^{OS}(\omega)+I^{AS}(\omega) \tag{23}$$

Introducing Eqs. (20)–(22) into Eq. (23) and applying simple mathematical operations may be obtained the informational power characteristic of the additional subsystem improving the real functioning process:

$$\left|W^{AS}(j\omega)\right|^2=\frac{S_{\overset{o}{H^{*''}}\overset{o}{H^{*''}}}(\omega)\cdot\left[S^2_{\overset{o}{H'}\overset{o}{H^{*''}}}(\omega)+S_{\overset{o}{H''}\overset{o}{H''}}(\omega)\cdot S_{\overset{o}{H'}\overset{o}{H'}}(\omega)-S^2_{\overset{o}{H'}\overset{o}{H''}}(\omega)\right]}{S_{\overset{o}{H'}\overset{o}{H'}}(\omega)\cdot S^2_{\overset{o}{H''}\overset{o}{H^{*''}}}(\omega)} \tag{24}$$

In Eq. (24) the spectral density $S_{\overset{o}{H''}\overset{o}{H^{*''}}}(\omega)$ is calculated in the way similar to Eqs. (9), (11), (13), (15), (17), that is given above. The analytical description of this spectral density has the following form:

$$S_{\overset{o}{H''}\overset{o}{H^{*''}}}(\omega)=0.1725\cdot\left[\frac{0.055}{0.055^2+(\omega+0.65)^2}+\frac{0.055}{0.055^2-(\omega+0.65)^2}\right] \tag{25}$$

The results of calculation of the informational power characteristic (Eq. (24)) of the additional subsystem in the frequency domain are shown in Fig. 14.

From the point of view of the informational quantum criterion in the frequency domain, the power characteristic of the real functioning system (OS) should be augmented by the power characteristic of the additional subsystem. Then, the output concentration from the system containing the subsystems (OS) and (AS) is equal to the required concentration of the phosphorus from the system (NS). Thus, the improvement of the quality of the phosphorus reduction process by using the addi-

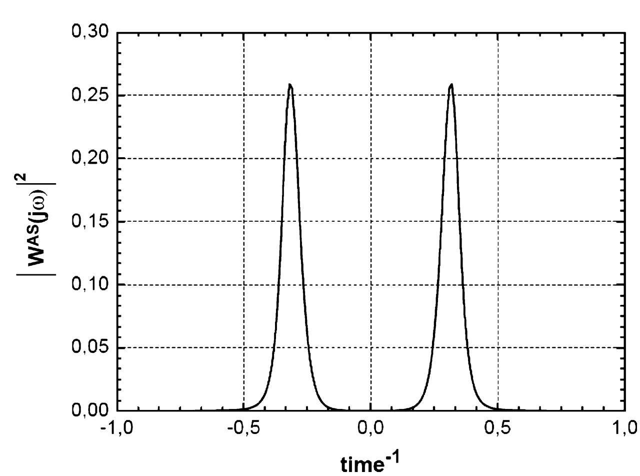

Fig. 14. The graphical presentation of the criterion Eq. (24).

tional precipitation process in a primary settling tank may be recommended.

4. Conclusions

The principles of treatment of the experimental data on phosphorus concentration reduction obtained at the industrial sewage treated plant based on the information theory were developed. It is shown that the informational theory characteristics may be used as the effective mathematical tool to the modelling and the simulation of the chemical engineering processes. The mathematical description should be supported by the correlation and spectral density analysis with the informational entropy as a state variable. The process of identification of the system is the information problem and the mathematical modelling should be undoubtedly accurate and more comprehensible. Several important practical conclusions may be derived from the considerations given in this paper:

- the mathematical description of chemical engineering systems may be done using the analysis of the industrial database by means of the informational entropy,
- the accurate analyses of the dynamical behaviour of a functioning engineering system depends on the extensive industrial database,
- the considerable simplification in formulation and calculation of the informational criterion may be reached if the experimental database has the normal distribution,
- the new approach to the definition of the quantum information in the time domain Eq. (3) and the frequency domain Eq. (4) is proposed,
- the improvement of the quality of the phosphorus reduction process in the raw sewage may be done by means of the additional precipitation process in a preliminary settling tank,
- the informational quantum criterion may be applied to the qualitative evaluation of the process efficiency in the chemical engineering systems.

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